Regular black holes from a nonlinear electrodynamics free from fractional powers of $F^{\mu\nu}F_{\mu\nu}$

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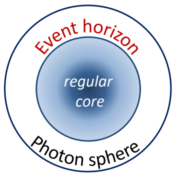
What is regular black hole (RBH)?

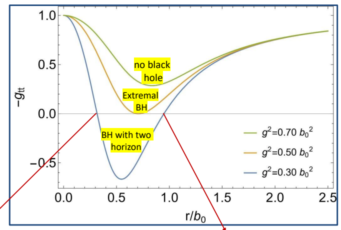
- Black holes in Einstein's GR have spacetime *singularities*.
- **Why is singularity a problem?**
  - I. Singular spacetimes are *not physical*.
  - II. Physical quantities *cannot be defined* at singularities.
  - III. Fate of gravitational collapse of *stable structure* is unknown.
- One possible remedy is regular black hole.



A typical RBH

Nature of the metric:

- $g^2 = 0$, *singular metric*
- $r \to 0$, a de-Sitter core
- $r \to \infty$ *asymptotically flat*
- $g^2 < 0.5\, {b_0}^2$, Double horizon
- $g^2 = 0.5\, {b_0}^2$, Single horizon
- $g^2 > 0.5\, {b_0}^2$, Horizon-less



Shortcomings of the existing models RBHs

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{\kappa} + L(\boldsymbol{F})\right), \qquad \boldsymbol{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Spherically symmetric	$-g_{tt} = g^{rr}$	$L(\boldsymbol{F})$
Bardeen's metric	$1 - \frac{2Mr^2}{(r^2+g^2)^{3/2}}$	$-\frac{(c\boldsymbol{F})^{5/4}}{g^2(1+\sqrt{c\boldsymbol{F}})^{5/2}}$
Hayward's metric	$1 - \frac{2Mr^2}{r^3+g^3}$	$-\frac{(c\boldsymbol{F})^{3/2}}{g^2(1+(c\boldsymbol{F})^{3/4})^2}$

- $F_{\theta\phi} = -q_m \sin\theta$ (*magnetic monopole*), $\boldsymbol{F} = \frac{{q_m}^2}{2r^4}$ (*singular at $r \to 0$*)

Problems of the matter terms $L(\boldsymbol{F})$

- $L(\boldsymbol{F})$ has **fractional power** of '$\boldsymbol{F}$'
  - I. Restricted to magnetic field only, in nature *negative F exists*
  - II. Gravitational coupling with *electric source not doable*
- Original objectives of NLE are not fulfilled
  - I. Removal of matter field singularities
  - II. Finiteness of *self-energy of a point charge*

Our objective

- Construct a RBH with $L(\boldsymbol{F})$ *free from fractional power* of $\boldsymbol{F}$
- Original aims of NLE are met

The new metric and its matter source

$$L(\boldsymbol{F}) = \frac{\gamma(3\eta\boldsymbol{F} - 1)\boldsymbol{F}}{(1+\eta\boldsymbol{F})^2}$$

No fractional power !!

- The magnetic solution:

$$g_{\mu\nu} \equiv \begin{pmatrix} -1 + \frac{{b_0}^2 r^2}{r^4+g^4} & 0 & 0 & 0 \\ 0 & \left(1 - \frac{{b_0}^2 r^2}{r^4+g^4}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}; \quad F_{\theta\phi} = q_m \sin\theta$$

- **Curvature invariants:**

$$g_{\mu\nu}R^{\mu\nu} = \frac{4b_0^2(3g^8 - 5g^4r^4)}{(r^4+g^4)^3}, \qquad R_{\mu\nu}R^{\mu\nu} = \frac{4b_0^4(9g^{16} - 14g^4r^{12} + 74g^8r^8 - 30g^{12}r^4 + r^{16})}{(r^4+g^4)^6}$$

$$R_{\mu\nu\sigma\delta}R^{\mu\nu\sigma\delta} = \frac{8b_0^4(3g^{16} - 10g^{12}r^4 + 74g^8r^8 - 34g^4r^{12} + 7r^{16})}{(r^4+g^4)^6}$$

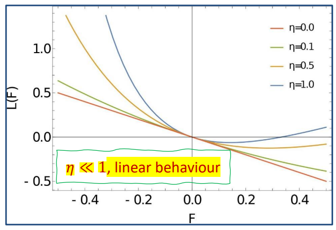
Flat spacetime analysis of matter term

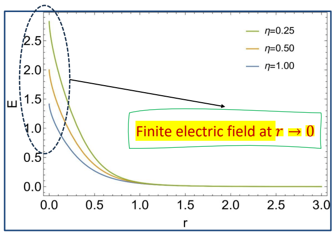
- Weak field limit, $L(\boldsymbol{F}) \approx -\gamma\boldsymbol{F} + 5\gamma\eta\boldsymbol{F}^2 - 9\gamma\eta^2\boldsymbol{F}^3 + \gamma O(\eta^3)$ — Similar to Born-Infeld model !!
- **Flat space equation of motion:**

$$\vec{\nabla}\cdot\vec{D} = 0, \quad \frac{\partial\vec{D}}{\partial t} - \vec{\nabla}\times\vec{H} = 0$$
$$\vec{\nabla}\cdot\vec{B} = 0, \quad \frac{\partial\vec{B}}{\partial t} + \vec{\nabla}\times\vec{E} = 0$$

- $\vec{D} = \frac{\partial L}{\partial\vec{E}} = \epsilon_i^j E_j, \quad \vec{H} = -\frac{\partial L}{\partial\vec{B}} = (\mu^{-1})_i^j B_j$

Nonlinearity of the Lagrangian is encoded via an '*anisotropic medium*'





- **Electric field due to a point charge:**
  - $\vec{\nabla}\cdot\vec{D} = e\,\delta(r)$,
  - $E + \frac{7}{2}\eta E^3 = \frac{e}{4\pi\gamma r^2}\left(1 - \frac{\eta}{2}E^2\right)^3$

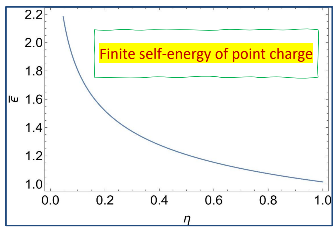
$$E_{max} = \sqrt{2/\eta}$$

- **Energy of a point charge:**

$$T^H_{\mu\nu} = -\frac{2}{\sqrt{-g}}\left(\frac{\partial(\sqrt{-g}L(F))}{\partial g^{\mu\nu}}\right)\Bigg|_{g=\eta}$$

$$\rho = -T^t_t$$

- $L(\boldsymbol{F})$ shows vacuum birefringence
- Causality and unitarity conditions are upheld.

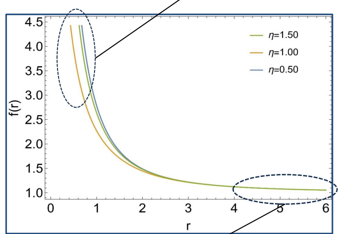


The electric solution

- Einstein's equation for point charge source,

$$\frac{f'}{r} + \frac{f-1}{r^2} = \frac{\gamma\kappa E^2(4+3\eta E^2(8+\eta E^2))}{(-2+\eta E^2)^3}$$

- $f(r) = -g_{tt} = g^{rr}$
- No horizon
- $g_{\mu\nu}R^{\mu\nu} = -\frac{8\gamma\kappa\eta E^4(10+3\eta E^2)}{(-2+\eta E^2)^3}$
- Divergence at $E = \sqrt{2/\eta}$ or, $r = 0$
- **A naked singularity**



Conclusion

Source field	Spacetime solution
Magnetic monopole (singular)	Regular Black hole (non singular)
Electric charge (non singular)	Naked singularity (singular)

Simultaneous resolution of field and spacetime singularities is a great achievement

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Journal reference

arXiv reference